

Course Brochure

Coordinate Geometry for JEE Main and Advanced

[With focus on Problem Solving Skills]

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[B.Tech. IIT Kanpur. 20+ Yrs, Exp]

? If $4a^2 + 9b^2 - c^2 + 12ab = 0$, then find the coordinates of possible points where the family of straight lines $ax + by + c = 0$ concurrent.
[Ans: $(-2, -3), (2, 3)$]

$(2a)^2 + (3b)^2 + 2(2a)(3b) - c^2 = 0$
 $(2a + 3b)^2 - c^2 = 0$
 $\Rightarrow (2a + 3b - c)(2a + 3b + c) = 0$
 $2a + 3b - c = 0$
 $\begin{cases} (-2)a + (-3)b + c = 0 \\ 2a + 3b + c = 0 \end{cases}$
 $a = -2, b = -3, c = 0$
 $(-2, -3)$

? Try Yourself
A circle C touches the line $x = 2y$ at the point $(2, 1)$ and intersects the circle $C_1: x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1 . Then the diameter of C is
[JEE (Main)-2021]
(1) $\sqrt{285}$ (2) $\sqrt{285}$
(3) $4\sqrt{15}$ (4) $4\sqrt{15}$

? If the equation $m^2(x+1) + m(y-2) + 1 = 0$ represents a family of lines, where 'm' is parameter then find the equation of the curve to which these lines will always be tangents.

$y = mx + \frac{1}{m}$
 $\frac{dy}{dx} = m$
 $\Rightarrow y^2 - 4x = 0$
 $(y-2)^2 - 4(x+1) = 0$
 $(y-2)^2 = 4(x+1)$

Property-3

The teacher

Yogesh Goyal is an IIT Kanpur alumnus (B. Tech.) with over two decades of experience in teaching, academic planning, and content development for competitive examinations such as JEE and NEET. He has worked in senior academic roles with reputed institutions across India and has been deeply involved in student mentoring, faculty training, national-level content creation, and the design of large-scale academic programs. He formally stepped away from institutional roles in June 2025 to focus entirely on building EduCanvas.



His work has always gone beyond classroom teaching. With a strong interest in visual pedagogy, data visualization, and structured content design, he has consistently explored how learning becomes clearer when concepts are supported by intelligent visuals, precise sequencing, and reduced cognitive load. This combination of subject expertise and learning design thinking forms the foundation of his academic philosophy.

EduCanvas represents his vision of transforming recorded education from simple video delivery into a scientifically designed learning system. He believes that great teaching achieves its full impact only when supported by strong learning architecture. Through EduCanvas, he aims to create courses where clarity, structure, and cognitive efficiency guide every aspect of the student's learning experience.

The Course

Coordinate Geometry is one of the most scoring and conceptually important areas of JEE Main and JEE Advanced. A strong command over this section forms the foundation for solving a wide range of problems involving straight lines and conic sections with speed, accuracy, and confidence.

The **Coordinate Geometry** course by **EduCanvas** is a comprehensive learning system that combines **Straight Lines, Circles, Parabola, Ellipse, and Hyperbola** into one structured program. Rather than treating these chapters as isolated topics, the course develops them as interconnected concepts, helping students build a deep and unified understanding of coordinate geometry.

Designed specifically for recorded learning, every concept is presented through carefully planned visual explanations, animated diagrams, structured problem-solving, and extensive guided practice. Students learn not only how to apply formulas but also the geometric intuition behind them, enabling them to solve JEE-level problems with greater confidence and efficiency.

Duration: 68 Hrs

Number of Lecture Practice Sheets: 46

Number of problems for Practice (with solutions): 930

Number of problems/examples solved fully by teacher: 363

Number of problems explained by teacher and left to students to get final answer i.e. Try Yourself: 220

How the Course Is Structured

The course consists of **32 comprehensive lectures**; each recorded with a duration of about **2 to 3 hours duration**. For better learning and easier viewing, each lecture is presented in the **EduCanvas App in two-three parts of approximately one hour each**.

This structure allows students to learn in focused sessions while maintaining continuity of the concept flow.

The lectures cover the complete theory of Straight Lines, Circles, Parabola, Ellipse and Hyperbola required for **JEE Main and Advanced**, beginning from geometric foundations and gradually progressing to advanced topics in each topic

Teaching Approach

All lectures are taught on a **digital board using carefully designed animated PPTs** that help students visualize mathematical ideas clearly.

Instead of long static writing on the board, concepts are explained using:

- structured diagrams
- step-by-step visual development
- animated illustrations
- clear highlighting of key ideas

This visual approach helps students **understand concepts faster and remember them longer**.



During each lecture, around **12–20 carefully selected illustrations** are solved completely, allowing students to see how concepts are applied in real problems.

In addition, **10-12 problems in every lecture are explained partially**, where the approach and strategy are discussed and students are encouraged to complete the solution themselves. This ensures that students actively engage with the concept rather than passively watching the lecture.

Lecture Practice Problems (LPP)

Practice is an essential part of mastering coordinate geometry. For this reason, every lecture is accompanied by **Lecture Practice Problems (LPP)**.

Each LPP contains approximately **20–25 carefully selected problems** based on the concepts covered in that lecture.

These problems help students:

- strengthen conceptual understanding
- develop problem-solving strategies
- build exam-level confidence

Detailed **solutions to all LPP questions are provided separately in PDF format**. Both the problem set and the solution PDF are available inside the **EduCanvas App** for easy access.



Hyperbola: LPP 1B

Content

1. Hyperbola as locus of point
 - a. using its focal-directrix property.
 - b. Two alternate definitions of Hyperbola.
2. Hyperbola in
 - a. standard positions (when centre is at origin and axes lies along x or y axis).
 - b. $\pm$ shifted positions

Level I

1. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is [JEE (Main)-2016]
(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{3}$ (4) $\frac{4}{3}$
2. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval [JEE (Main)-2019]
(1) (2,3) (2) (3/2,2) (3) (1.3/2) (4) (3, ∞)
3. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is [JEE (Main)-2019]
(1) $\frac{5}{2}$ (2) $\sqrt{3}$ (3) $\frac{3}{\sqrt{2}}$ (4) 2
4. Let $S = \{(x, y) \in \mathbb{R}^2; \frac{x^2}{1+r} - \frac{y^2}{1-r} = 1\}$, where $r \neq \pm 1$. Then S represents [JEE (Main)-2019]
(1) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$
(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$
(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$
(4) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$
5. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is [JEE (Main)-2019]
(1) 2 (2) $\frac{13}{5}$ (3) $\frac{13}{6}$ (4) $\frac{13}{12}$
6. If the vertices of a hyperbola be at (-2,0) and (2,0) and one of its foci be at (-3,0), then which one of the following points does not lie on this hyperbola? [JEE (Main)-2019]
(1) (4, $\sqrt{15}$) (2) (6.5, $\sqrt{2}$) (3) (2, $\sqrt{6}$, 5) (4) (-6.2, $\sqrt{10}$)

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Parabola: LPP 3B

Content

1. Tangents to parabola
 - i. Equation of tangent at a point on the parabola.
 - a. In terms of parametric coordinates
 - b. In terms of slope.
2. Tangents passing through given point outside a parabola and their point of tangency.

Level I

1. Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.
Statement-I: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.
Statement-II: If the line, $y = mx + \frac{\sqrt{5}}{m}$ ($m \neq 0$) is their common tangent, then m satisfies $m^4 - 3m^2 - 2 = 0$. [JEE (Main)-2013]
(1) Statement-I is true; statement-II is true; statement-II is a correct explanation for statement-I
(2) Statement-I is true; statement-II is true; statement-II is not a correct explanation for statement-I
(3) Statement-I is true; statement-II is false
(4) Statement-I is false; statement-II, is true
2. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is [JEE (Main)-2014]
(1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{2}$
3. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1,2) and the x-axis is [JEE (Main)-2019]
(1) $4\pi(3 + \sqrt{2})$ (2) $8\pi(2 - \sqrt{2})$ (3) $8\pi(3 - 2\sqrt{2})$ (4) $4\pi(2 - \sqrt{2})$
4. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then |c| is equal to [JEE (Main)-2019]
(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 2 (4) $\sqrt{2}$
5. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to [JEE (Main)-2020]
(1) -64 (2) 128 (3) -32 (4) -128
6. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A(\frac{1}{2}, -2)$, then the equation of the tangent to it at B is [JEE (Main)-2020]
(1) $x - 2y + 8 = 0$ (2) $x + 2y + 8 = 0$ (3) $2x - y - 24 = 0$ (4) $2x + y - 24 = 0$

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Topics Covered in the Course

The complete lecture plans for each topic are given below

Straight Lines

Lecture 1

1. Distance formula
2. Section formula:
 - i. internal and external division
3. Centres of triangle
 - i. Centroid,
 - ii. Incentre & Ex-centres
 - iii. Orthocentre
 - iv. Circumcentre
4. Area of triangle
 - i. Area of any polygon.
 - ii. Area = 0 implies collinearity.

Lecture 2

1. Concept of locus
2. Inclination and slope of line
3. Equation of line when different type of information is available about line
 - i. Two points on a line are known
 - ii. Intercepts (?) on x and y axis are known
 - iii. Slope of line and a point on it is known
 - iv. General form of equation of line
4. Inter conversion in different forms (wherever possible)
5. Point of intersection and concurrency of three lines
6. Angle between two lines, conditions for two lines being parallel and perpendicular.

Lecture 3

1. Equation of line (special cases)
 - i. Normal form / conversion to this form
 - ii. Parametric form (Problems involving distances on a line from a point on that line)
 - iii. Equation of line in determinant form
2. Two lines passing through a given point making equal angle with a given line (there slopes and hence equation.
3. Foot of perpendicular & Reflection
4. Position of points w.r.t. a given line
 - i. Point lies inside a triangle or any given polygon
 - ii. Foot of perpendicular and reflection of a point in given line

Lecture 4

1. Perpendicular distance of point from line
2. Bisectors of angle between two lines
 - i. Identifying the bisector of part containing origin or any given point in general
 - ii. Identifying the bisectors of acute or obtuse angle between lines precisely
 - iii. Bisectors expressed in parametric form
 - iv. Identifying the bisectors of angles of a triangle i.e. internal or external, precisely

3. Family of lines
 - i. Parallel and perpendicular to given line
 - ii. Passing through point of intersection of two given lines
 - iii. Miscellaneous Families (getting diagonals of a parallelogram)

Lecture 5

1. General equation of Pair of lines
 - i. Passing through origin or in general (Second degree homogeneous or non-homogeneous equations i.e., SDHE & SDNHE)
 - ii. Condition for SDHE & SDNHE to represent pair of lines
 - iii. Method to separate the line in both cases (in general avoid it)
 - iv. Finding point of intersection w/o separating the lines
2. Relations between slopes of lines and coefficients of SDHE or SDNHE
 - i. Angle between pair of lines
 - ii. Conditions for being parallel/coincident and perpendicular

Lecture 6

1. Combined equation of bisectors of angle between the lines
2. Concept of homogenization and its applications in problems
3. Transformation of axis
 - i. Shifting origin but no rotation (useful)
 - ii. Rotation of axis (rarely used)

Circles

Lecture 01

1. Brief review of pure Geometry involving circles
 - i. Geometrical properties of circle
 - ii. Properties involving tangents to a circle
 - iii. Properties of cyclic quadrilaterals.
2. Definition of circle as locus in two ways
 - i. locus of a point which is always at fixed distance from a given point
 - ii. Apollonius definition with its proof.
 - iii. Finding equation of circle when its center & radius is known with stress on using pure geometry in finding center & radius.

Lecture 02

1. Equation of circle
 - i. In general form from center –radius definition
 - ii. Equation of circle in diametric form
 - iii. Parametric equation of circle.
 - iv. Equation of chord joining two points on circle in parametric form.

Lecture 03

1. Position of a point w.r.t. a circle (inside, outside or on).
2. Intercepts
 - i. Position of Axes w.r.t. given circle
 - ii. Position of any line w.r.t. circle
 - iii. lengths of intercepts made on axes & in general on any line
 - iv. Chord with a given point as mid-point

Lecture 04

1. Tangents at a point on circle
 - i. Concept of tangent to a general curve.
 - ii. Equation of tangent at a given point on a circle (justify $T = 0$)
 - iii. Tangent with a given slope & its general equation for circle with center at origin
 - iv. Tangent at point in parametric form
 - v. Point of intersection of two tangents
2. Tangents from outside
 - i. Two tangents from a point outside the circle & finding their equations
 - ii. Finding point of tangency of any tangent
 - iii. Length of tangents
 - iv. Power of a point

Lecture 05

1. Tangents from outside
 - i. Chord of contact
 - ii. Various results and important points regarding the figure formed by the two tangents from a point & their chord of contact
 - iii. Combined equation of two tangents from a point outside the circle
 - iv. Director circle and similar locus based on angle between tangents

Lecture 06

1. Family of circles
 - i. passing through point of intersection of two circles
 - ii. passing through point of intersection of a circle and a line
 - iii. passing through two given points
 - iv. touching a given line at a given point on it
 - v. miscellaneous families

Lecture 07

1. Circles and common tangents
 - i. Relative positions of two circles
 - ii. Number of common tangents to two circles
 - iii. Finding the equations of common tangents
 - iv. Finding the lengths of common tangents.

Lecture 08

1. Radical axes / common chord
 - i. its location & other properties. Radical centre.
2. Angle of intersection of two curves in general and circle as a special case.
 - i. Orthogonality of two circles.
 - ii. Orthogonality of three circles

Parabola

Lecture 01

3. Introduction
 - i. Conic sections as the curves obtained by cutting a cone by a plane at different angles.
 - ii. Conic sections as locus of point using their focal-directrix property, covering both the cases when focus lies on the directrix and when it is separated.

4. Parabola in standard and shifted positions
 - i. when vertex is at origin and axis lies along x or y axis
 - ii. when parabola has its vertex at (a, b) and axis is parallel to x or y axis
5. Finding the components of parabola when its equation is given
6. Finding the equation when some of its components are given.

Lecture 02

2. Parametric coordinates
3. Equation of chord joining two points in parametric form & hence property of focal chord.
4. Position of a point w.r.t. parabola.
5. Position of line w.r.t. parabola.
6. Parabola and chords
 - i. length of intercept made by parabola on a line.
 - ii. length of focal chord.
 - iii. focal distance of any point on parabola

Lecture 03

3. Tangents to parabola
 - i. Equation of tangent at a point on the parabola.
 - ii. In terms of parametric coordinates
 - iii. In terms of slope.
4. Tangents passing through given point outside a parabola and their point of tangency.
5. Problems on using the equations of tangents in different forms.

Lecture 04

2. Combined equation of pair of tangents from a point outside a parabola.
3. Properties related to tangents to a parabola.

Lecture 05

2. Chord of contact, its equation and length.
3. Equation of normal at a point on parabola.
 - i. Equation of normal in parametric form.
 - ii. Equation of normal if slope is known/given.
4. problems on using the equations of normals in different forms.

Lecture 06

2. Properties related to normals & tangents.
3. conormal points etc.
4. System of parallel chords. concept of Diameter and hence centre.
5. Chord with a given point as mid-point.

Ellipse

Lecture 01

1. Ellipse as locus of point
 - a. using its focal-directrix property.
 - b. Two alternate definitions of ellipse.

2. Ellipse in
 - a. standard positions (when centre is at origin and axes lies along x or y axis).
 - b. Ellipse in 2 shifted positions (when Ellipse has its vertex at (a, b) and axes are parallel to x or y axis).
3. finding the components of Ellipse when its equation is given
4. Finding the equation when some of its components are given.

Lecture 02

1. Radius vector of any point on ellipse.
2. Auxiliary circle, eccentric angle and hence Parametric coordinates of any point on Ellipse in standard and shifted position.
3. Position of a point w.r.t. Ellipse
4. Ellipse and chords
 - a. Equation of chord joining two points, in parametric form.
 - b. Focal chord and properties related to it. (2 Properties)
 - c. Position of line w.r.t. Ellipse
 - d. length of intercept made by Ellipse on a line.

Lecture 03

1. Equation of tangent at a point on the Ellipse.
 - a. Cartesian form
 - b. Parametric coordinate form.
 - c. Equation of a tangent if its slope is known.

Lecture 04

1. Two tangents passing through given point outside an ellipse
 - a. their equations
 - b. points of tangency.
2. Combined equation of pair of tangents from a point outside an ellipse.
3. Director Circle.
4. Properties related to tangents to an ellipse (5 Properties)

Lecture 05

1. Chord of contact, its equation and length.
2. Equation of normal at a point on ellipse.
 - a. Cartesian form
 - b. In parametric form.
 - c. In slope form

Lecture 06

1. Properties related to normals (2 properties)
2. conormal points etc.
3. System of parallel chords.
 - a. Concept of Diameter and hence centre.
 - b. Chord with a given point as midpoint.
 - c. 2 properties

Hyperbola

Lecture 01

1. Hyperbola as locus of point
 - a. using its focal-directrix property.
 - b. Two alternate definitions of Hyperbola.
2. Hyperbola in
 - a. standard positions (when centre is at origin and axes lies along x or y axis).
 - b. Hyperbola in 2 shifted positions (when Hyperbola has its vertex at (a, b) and axes are parallel to x or y axis).
3. finding the components of Hyperbola when its equation is given
4. finding the equation when some of its components are given.

Lecture 02

1. Radius vector of any point on Hyperbola.
2. Auxiliary circle, eccentric angle and hence Parametric coordinates of any point on Hyperbola in standard and shifted position.
3. Position of a point w.r.t hyperbola and chords
 - a. Equation of chord joining two points, in parametric form.
 - b. Focal chord and properties related to it.
 - c. Position of line w.r.t. Hyperbola.
 - d. Length of intercept made by Hyperbola on a line.
4. Equation of tangent at a point on the Hyperbola.
 - a. Cartesian form
 - b. Parametric coordinate form.
 - c. Slope form
5. Two tangents passing through given point outside a Hyperbola
 - a. Their equations
 - b. points of tangency.
6. Director Circle.

Lecture 03

1. Combined equation of pair of tangents from a point outside a Hyperbola.
2. Chord of contact
 - a. its equation and length.
3. Equation of normal
 - a. at a point on Hyperbola.
 - b. in parametric form.
 - c. Equation of normal if slope is known/given.
4. Conormal points

Lecture 04

1. Properties related to tangents and normals to a Hyperbola.
2. conormal points etc.
3. System of parallel chords.
 - a. concept of Diameter and hence centre.
 - b. Chord with a given point as midpoint.

Lecture 05

1. Asymptotes to a Hyperbola.

2. Equation of a Hyperbola, it's Asymptotes and its's Conjugate Hyperbola.
3. Properties related to asymptotes.

Lecture 06

1. Rectangular Hyperbola.
 - a. Equations of Rectangular Hyperbola referred to its asymptotes as its axes.
 - b. Parametric coordinates on rectangular hyperbola.
 - c. equations of tangent & Normals to rectangular Hyperbola.
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Who This Course Is For

This course is ideal for students who:

- are preparing for **JEE Main and JEE Advanced**
 - want **strong conceptual clarity in coordinate geometry**
 - prefer **visual explanations rather than rote formula learning**
 - want **structured practice along with theory**
-

What Students Gain from This Course

By the end of the course, students will be able to:

- understand Coordinate Geometry concepts deeply
- approach problems using geometric insight
- solve exam-level questions with confidence
- connect concepts of Coordinate Geometry with other topics of Mathematics

Some Screenshots

? Find the equation of the locus of a point P moving in a plane such that its distance from a fixed point (a, b) in the same plane is always equal to 2

$P: (h, k)$

Hyperbola: $e > 1$

$PF_1/PF_2 = 11.7/10 = e$

Important Point
The incentre of the orthic triangle is orthocentre of the given triangle

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $b^2 = a^2(e^2 - 1)$

Foci: F_1, F_2 Directrices

Equation of transverse axis: $y = 0$

Length of the transverse axis: $2a$

Vertices: End points of transverse axis

Equation of conjugate axis: $x = 0$

? A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P , that divides the rod AB internally in the ratio $2:1$ is $9(x^2 + cy^2 + 2xy + 4y + 28y) - 76 = 0$, then $c - b - y$ is equal to:

(1) 23 (2) 21 (3) 21 (4) 22

[JEE Main 2021]

? Let a point P be such that its distance from the point $(-5, 0)$ is thrice the distance of P from the point $(5, 0)$. If the locus of the point P is a circle of radius r , then $(4r^2)$ is equal to (where $[-]$ represents greatest integer function).

[JEE (M) 2011]

? C_1 and C_2 are two fixed circles with radii r_1 and r_2 such that C_1 is contained in C_2 . A third circle C moves in such a way that it touches C_2 internally and C_1 externally, then the locus of its centre is

(A) Circle (B) Parabola (C) Ellipse (D) Straight line

? Let R be the point $(3, 7)$ and let P and Q be two points on the line $x + y = 5$ such that $\triangle PQR$ is an equilateral triangle. Then the area of $\triangle PQR$ is

(1) $\frac{25}{\sqrt{3}}$ (2) $\frac{25\sqrt{3}}{2}$ (3) $\frac{25}{2}$ (4) $\frac{25}{\sqrt{3}}$

[JEE (Main)-2021]

Alternate Method to define Ellipse

Point moving in such a way that sum of its distances from two foci is always a constant

? Multiple Options Correct

Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

[IIT-JEE-2011 P1]

The eccentricity of the hyperbola is $\frac{\sqrt{5}}{2}$

The hyperbola is $x^2 - 3y^2 = 3$

$F_{hyperbola} = (\pm a, 0) \in \{(1, 2)\}$

? If $3x + 2y + 6c = 0$ the family of straight lines $ax + by + c = 0$ passes through a fixed point. Find the coordinates of the fixed point.

? Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A . Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A , then the area of the triangle ABC is

(A) $1 - \frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2} - 1$ (C) $1 + \frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{2} + 1$

[IIT JEE 2008 P2]

$y^2 - 4ax - 2\beta y + 4a + \beta^2 = 0$

$\frac{(x-a)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$

$y^2 = 4a$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

? $(3.38) / (4.54) = 0.75$

Ellipse: $e < 1$

$(3.79) / (5.08) = 0.75$

Property-2

Hyperbola: Oblique position

$\frac{PM^2}{a^2} - \frac{PN^2}{b^2} = 1$

$\frac{(Distance from C_1 A)^2}{(semi T.A.)^2} - \frac{(Distance from F_1 A)^2}{(semi C.A.)^2} = 1$

Ellipse: $e < 1$

$(3.38) / (4.54) = 0.75$

$(3.79) / (5.08) = 0.75$

Auxiliary circle and parametric coordinates

The circle, which is described on the major axis, A_1A_2 of hyperbola as diameter, is called the auxiliary circle of 'hyperbola'.

? If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the midpoints of the tangents intercepted between the coordinate axes lie on the curve

[JEE (Main)-2019]

(1) $\frac{1}{x} + \frac{1}{y} = 1$ (2) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (3) $\frac{1}{2x} + \frac{1}{4y} = 1$ (4) $\frac{1}{2x} + \frac{1}{4y} = 1$

? Tangents are drawn to a unit circle with centre at origin from every point on the line $2x + y = 4$.

(i) The chords of contact pass through a fixed point, find its coordinates.

$[Ans: (2, \frac{1}{2})]$

$\frac{x^2}{1} + \frac{y^2}{1} = 1$

$2x + y = 4$

Position of line w.r.t. to hyperbola

Circle orthogonal to 3 given circles

Circle whose centre is the radical centre of 3 given circles is orthogonal to length of tangent drawn from radical centre to any one of the circles.

Tangent as a limiting case of secant

?

Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is [JEE (Adv)-2015 (P1)]

Try Yourself?

If, through the vertex O of a parabola $y^2 = 4x$ chords OP and OQ are drawn at right angles to one another, then for all position of P, Q cuts the axis of the parabola at a fixed point. Find the value of p . [Modified, IIT-JEE-1994]

$y(t_1 + t_2) = 2(x_1 + x_2)$

Constant angle between tangents

?

The locus of the mid-points of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is [JEE (Main)-2021]

(1) $x^2 + y^2 = 25$
(2) $x^2 + y^2 + 144y^2 = 0$
(3) $x^2 + y^2 - 2x^2 - 16y^2 = 0$
(4) $x^2 + y^2 + 2x^2 + 16y^2 = 0$

Try Yourself?

A tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at P and Q . Find the locus of the midpoint of PQ . [Ans: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$]

?

If the equation $m^2(x+1) + m(y-2) + 1 = 0$ represents a family of lines, where 'm' is parameter then find the equation of the curve to which these lines will always be tangents.

Property-3

Try Yourself?

The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{5}{4}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x - and y -axes are a and b respectively, then $|6a| + |5b|$ is equal to [JEE (Main)-2021]

?

Find the equation to the locus of the middle point of the chord of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which subtends right angle at a given point $A(a, b)$.

Multiple Options Correct

Tangents are drawn to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$, parallel to the straight line $2x - y = 1$. The point of contact of the tangents on the hyperbola are [IIT-JEE-2012 (P1)]

(A) $(\frac{2}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$ (B) $(-\frac{2}{\sqrt{2}}, \frac{3}{\sqrt{2}})$ (C) $(3\sqrt{3}, -2\sqrt{2})$ (D) $(-3\sqrt{3}, 2\sqrt{2})$

$544c = 2$

$y = 2x \pm \sqrt{3} \sqrt{1 - \frac{4}{9}}$
 $2x \pm \frac{\sqrt{5}}{3}$
 $(\frac{2}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$
 $(-\frac{2}{\sqrt{2}}, \frac{3}{\sqrt{2}})$

Property-3

Property 3-2

The tangents at the extremities of a chord intersect at right angles on the directrix.

?

A common tangent T to the curves $C_1: \frac{x^2}{4} + \frac{y^2}{9} = 1$ and $C_2: \frac{x^2}{16} - \frac{y^2}{9} = 1$ does not pass through the fourth vertex of the rectangle formed by the tangents at $(2x_1, y_1)$ and C_2 at $(2x_1, y_1)$ is equal to [IIT-JEE-2022]

Try Yourself?

Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 8$. If S and S' denote the foci of the parabola, then PS lies on a ratio SS' in a ratio [JEE (Main)-2010]

(1) 13:5 (2) 14:13
(3) 5:13 (4) 2:1

$y = mx \pm \sqrt{16m^2 - 9}$
 $y = mx \pm \sqrt{16m^2 - 9}$

The pair of tangents from a point (x_1, y_1) outside the Hyperbola

$y - y_1 = \frac{k - y_1}{h - x_1}(x - x_1)$
 $y = \frac{k - y_1}{h - x_1}x + \frac{h y_1 - k x_1}{h - x_1}$

Property 3-3

The tangents at the extremities of a focal chord intersect at right angles on the directrix.

A circle on any focal chord as diameter touches the directrix.

Property-5

Feet of the perpendiculars from either focus on a variable tangent to an ellipse lies on its axis.

Property 5-1

?

Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact. [IIT JEE 2002]

$A(x_1, y_1)$ $C_1: \frac{x^2}{9} - \frac{y^2}{16} = 1$
 $C_2: x^2 + y^2 = 9$
 $\frac{x_1^2}{9} - \frac{y_1^2}{16} = 1$
 $x_1^2 + y_1^2 = 9$
 $\frac{x_1^2}{9} - \frac{y_1^2}{16} = 1$
 $x_1^2 + y_1^2 = 9$
 $\frac{x_1^2}{9} - \frac{y_1^2}{16} = 1$
 $x_1^2 + y_1^2 = 9$
 $\frac{x_1^2}{9} - \frac{y_1^2}{16} = 1$
 $x_1^2 + y_1^2 = 9$

? The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N , respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) Vertex is $(\frac{2a}{3}, 0)$ (B) Directrix is $x = 0$
 (C) Latus rectum is $\frac{2a}{3}$ (D) Focus is $(a, 0)$

[IIT-JEE-2009 P2]

? Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x -axis at $(9,0)$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

[IIT-JEE-2011 P2]

? Try Yourself
 Let the tangent drawn to the parabola $y^2 = 24x$ at the point (a, β) is perpendicular to the line $2x + 2y$ then the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at point $(a + 4, \beta + 4)$ does NOT pass through the

(A) $(5, 10)$ (B) $(20, 12)$
 (C) $(10, 8)$ (D) $(15, 13)$

[IIT-JEE-2022]

Number of normals
 In general, four normals can be drawn to a hyperbola from any point.
 A, B, C, and D are called conormals
 $\alpha + \beta + \gamma + \delta$ is an odd multiple of π

? Let $P(a, \beta)$ be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid-point is $(1, \frac{5}{2})$, then $(\alpha - 2\beta)(\beta - 8)$ is equal to.

[JEE Main 2021]

? Ellipse: $e < 1$
 $(3.38/14.64) = 0.75$
 $(5.23/7.02) = 0.75$

? Try Yourself
 Find the locus of the middle point of the chords of the parabola $y^2 = 4ax$ which are normal to the parabola

? Multiple Options Correct
 An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are also coordinate axes, then

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$
 (C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm \sqrt{2}, 0)$

[IIT-JEE-2009 P2]

? Let $E_1, E_2, E_3, \dots$ be a sequence of ellipses such that their centres and eccentricities are same as that of E_1 , and the length of minor axis of E_i is $\frac{1}{i}$ th of the length of the major axis of E_{i+1} ($i \geq 1$). If A_i is the area of the ellipse E_i , then $\sum_{i=1}^{\infty} A_i$ is equal to ...

[JEE Main 28 Jan M 2023]

Property-2

Property-2

Property-3
 The tangent & normal at a point P on the hyperbola bisect the external & internal angles between the focal distances of P .

? $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
 P in the first quadrant.
 Let $\angle POF = \alpha$
 1. $\angle POF = \alpha$
 2. $\angle POF = \alpha$
 3. $\angle POF = \alpha$
 4. $\angle POF = \alpha$

[JEE Adv 2022 P2]

System of parallel chords and diameter

System of parallel chords and diameter

? Try Yourself
 Tangents are drawn from any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the circle $x^2 + y^2 = r^2$, prove that the chords of contact are tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^4$.

? The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is

(1) $y^2(x^2 - 1) = x^2$ (2) $y^2(x - 2) = x^2$
 (3) $x = y^2$ (4) $x^2(x - 2) = y^2$

[JEE (Main)-2021]